How to uniquely determine your location in a graph? A metric dimension problem

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Abstract

The metric dimension problem was first introduced in 1975 by Slater [35], and independently by Harary and Melter [16] in 1976; however the problem for hypercube was studied (and solved asymptotically) much earlier in 1963 by Erdős and Rényi [10]. A set of vertices S *resolves* a graph G if every vertex is uniquely determined by its vector of distances to the vertices in S. The *metric dimension* of G is the minimum cardinality of a resolving set of G.

Garey and Johnson [13], and also Khuller *et al.* [23], showed that determining the metric dimension of an arbitrary graph is an NP-complete problem. The problem is still NP-complete even if we consider some specific families of graphs, such as planar graphs [9] or Gabriel unit disk graphs [18]. Thus research in this area are then constrained towards: characterizing graphs with particular metric dimensions, determining metric dimensions of particular graphs, and constructing algorithm that best approximate metric dimensions. Until today, only graphs of order n with metric dimension 1, $n-3$, $n-2$, and $n-1$ have been characterized [6, 21].

On the other hand, researchers have determined metric dimensions for many particular classes of graphs, such as trees [6, 16, 23], cycles [6], complete multipartite graphs [6, 31], grids [27], wheels [4, 5, 33], fans [5], unicyclic graphs[29], honeycombs [26], circulant graphs [30, 19], Jahangir graphs [36], and Sierpíski graphs [22]. Recently in 2011, Bailey and Cameron [2] established relationship between the base size of automorphism group of a graph and its metric dimension. This result then motivated researchers to study metric dimensions of distance regular graphs, such as Grassman [3, 15], Johnson, Kneser [1], and bilinear form graphs [12, 14]. There are also some results of metric dimensions of graphs resulting from graph operations, for instance: Cartesian product graphs [27, 23, 5], joint product graphs [4, 5, 33], corona product graphs [37, 20], lexicographic product graphs [32], and amalgamation product graphs [34].

In the area of constructing algorithm that best approximate metric dimensions, recently researchers have utilized integer programming [8], genetic algorithm [24], variable neighborhood search based heuristic [28], and greedy constant factor approximation algorithm [17].

An natural analogue for oriented graphs was introduced by Chartrand, Raines, and Zhang much later in 2000 [7]. Since a directed path from one vertex to another needs

not to exist, not every oriented graph has a dimension. Therefore one fundamental question is the necessary and sufficient conditions for the dimension to be defined and the answers are still unknown to date. Unlike the undirected version, there are not many results known for directed metric dimension. Characterization of oriented graphs with particular dimension is only known for dimension 1 [7]. Researchers have also studied the directed metric dimension of tournaments [25] and Cayley digraphs [11].

I this talk I will present a short historical account, known techniques, recent results, and open problems in the area of metric dimension for both undirected and oriented version.

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