Metric Dimension of Directed Graphs with Cyclic Covering

Rinovia Simanjuntak, Sigit Pancahayani, and Yozef G. Tjandra *

The metric dimension problem was first introduced in 1975 by Slater [6], and independently by Harary and Melter [4] in 1976; however the problem for hypercube was studied (and solved asymptotically) much earlier in 1963 by Erdős and Rényi [2]. A set of vertices S *resolves* a graph G if every vertex is uniquely determined by its vector of distances to the vertices in S. The *metric dimension* of G is the minimum cardinality of a resolving set of G.

An natural analogue for oriented graphs was introduced by Chartrand, Raines, and Zhang much later in 2000 [1]. Consider an oriented graph D. For a vertex v and an ordered set $W = \{w_1, w_2, \ldots, w_k\}$ of vertices, the k-vector $r(v|W) = (d(v, w_1), d(v, w_2), \ldots, d(v, w_k))$ is referred to as the *(directed) representation* of v with respect to W, where $d(x, y)$ denotes the directed distance from x to y. If $r(v|W)$ exists for every vertex v, then the set W is called a *resolving set* for D if every two distinct vertices have distinct representations. A resolving set of minimum cardinality is called a *basis* for D and this cardinality is the *(directed) dimension*, $dim(D)$, of D. An oriented graph of dimension k is also called k-dimensional.

Since not every oriented graph has a dimension, one fundamental question is the necessary and sufficient conditions for $dim(D)$ to be defined; and the answers are still unknown. Unlike the undirected version, there are not many results known for directed metric dimension. Characterization of k-dimensional oriented graphs is only known for $k = 1$ [1]. Researchers have also studied the directed metric dimension of tournaments [5] and Cayley digraphs [3].

Let G be a graph with cyclic covering. An orientation on G is called C_n-*simple* if all directed C_n in the oriented graph are strong. Here we study metric dimension of two simply oriented graphs: the wheels and the fans. In addition we prove a sharp upper bound for the metric dimension for arbitrary strong oriented graphs, which is an improvement of the bound provided in [1].

*Combinatorial Mathematics Research Group, Faculty of Mathematics and Natural Sciences, Institut Teknologi Bandung, Jl. Ganesa 10, Bandung 40132, Indonesia. E-mail: rino@math.itb.ac.id

References

[1] G. Chartrand, M. Raines, P. Zhang, The Directed Distance Dimension of Oriented Graphs, *Math. Bohemica*, **125** (2000), 155-168.

[2] P. Erdős and A. Rényi, On two problems of information theory, *Magyar Tud. Akad. Mat. Kutat Int. Kzl* **8** (1963), 229-243.

[3] Melodie Fehr, Shonda Gosselin, Ortrud R. Oellermann, The metric dimension of Cayley digraphs, *Discrete Math.* **306** (2006), 3141.

[4] F. Harary, and R.A. Melter, On the metric dimension of a graph, *Ars Combin.* **2** (1976), 191-195.

[5] Antoni Lozano, Symmetry Breaking in Tournaments, *Elec. Notes Discrete Math.* **38** (2011), 579584.

[6] P.J. Slater, Leaves of trees, *Congr. Numer.* **14** (1975) 549-559.