

METRIC DIMENSION OF GRAPHS: RECENT RESULTS AND OPEN PROBLEMS

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The metric dimension problem was first introduced in 1975 by Slater [12], and independently by Harary and Melter [6] in 1976; however the problem for hypercube was studied (and solved asymptotically) much earlier in 1963 by Erdős and Rényi [4]. A set of vertices S *resolves* a graph G if every vertex is uniquely determined by its vector of distances to the vertices in S . The *metric dimension* of G is the minimum cardinality of a resolving set of G .

Garey and Johnson [5] showed that determining the metric dimension of an arbitrary graph is an NP-complete problem. Thus research in this area are then constrained towards: characterizing graphs with particular metric dimensions, determining metric dimensions of particular graphs, and constructing algorithm that best approximate metric dimensions. Until today, only graphs of order n with metric dimension 1, $n - 3$, $n - 2$, and $n - 1$ have been characterized [2, 8, 11]. On the other hand, researchers have determined metric dimensions for many particular classes of graphs. In the area of constructing algorithm that best approximate metric dimensions, recently researchers have utilized integer programming [3], genetic algorithm [9], variable neighborhood search based heuristic [10], and greedy constant factor approximation algorithm [7].

Recently in 2011, Bailey and Cameron [1] established relationship between the base size of automorphism group of a graph and its metric dimension; this result then motivated researchers to study metric dimensions of distance regular graphs. There are also some results of metric dimensions of graphs resulting from graph operations.

In this talk I will present a short historical account, known techniques, recent results, and open problems in the area of metric dimension.

Keywords distance, resolving set, metric dimension.

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